

**YMC Management Review****Volume 17, No.1, 2024 pp. 23-31****An Application of Welfare Game Theory on Discussing the Reactions between Ukraine and US-NATO in the Russia-Ukraine War**

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**Abstract**

The Russia-Ukraine war broke out on February 24, 2022, shaking the international community. To understand the behavior of both Russia and Ukraine in this war, as well as how military and economic aid from the United States, the European Union, and NATO influences Ukraine's actions, this paper explores the interaction between Ukraine and the U.S.-EU alliance through the viewpoint of welfare game theory. The discussion focuses on how these interactions affect the progress of the Russia-Ukraine war. International political and military activities are based on national interests. The U.S.-EU alliance's support for Ukraine in resisting the Russian invasion must consider the mutual interests, losses, and related conditions between the U.S., EU, and Ukraine.

**Keywords:** *Welfare game theory, Russia-Ukraine war, Ukraine, U.S.-EU alliance*

## 1. Introduction

This study primarily employs game theory to explore the Russia-Ukraine war. In the context of economic and military asymmetry between the two sides, the United States and the European Union stepped forward to provide Ukraine with military aid, diplomatic mediation, and high-tech support, while imposing economic sanctions on Russia. These sanctions aimed to help Ukraine resist Russia's invasion, significantly restricting Russia's foreign trade. However, despite external support, Ukraine's economy, already in a fragile state, has been further devastated by the war. On the other hand, while the EU strongly supports Ukraine, its energy needs are still dependent on Russia. Clearly, the economies of these countries are interconnected and mutually constrained. Therefore, this study uses game theory to analyze the optimal decision strategies based on the positions and interactions of all parties involved. The paper focuses on the U.S.-European alliance's support for Ukraine in the Russia-Ukraine war and uses welfare game theory to deduce the most optimal strategies for both sides.

Hsieh (2022) noted that after several months of conflict, the war between Russia and Ukraine has yet to yield decisive results, and there is a lack of consensus for negotiations. Western countries continue to provide aid to Ukraine and impose economic sanctions on Russia. Given the current situation, this has turned into a form of "proxy war," making a quick resolution unlikely and leading instead to a protracted war of attrition. This study attempts to analyze the ongoing support from the U.S. and European allies for Ukraine's logistics and supplies in its continued conflict with Russia through the welfare game model. In this relationship, the U.S.-EU alliance, which provides resources, hopes Ukraine will act in line with their expectations—namely, weakening Russia's economic and military capabilities to remove its status as a military power. Meanwhile, Ukraine, the recipient of these resources, seeks to meet the expectations of the U.S.-EU alliance and strengthen its economic and military alliances with the Western world. Through the interaction of welfare games, this paper aims to explore the optimal decision strategies for both sides.

## 2. Methodology

Welfare game theory is based on the relationship between two parties in the game, where "Party A gives, and Party B receives." In this game, Party A has abundant resources, and Party B desires to obtain these resources. Party A (giver) hopes that Party B's (receiver) actions align with its expectations, while Party B seeks to act in accordance with Party A's expectations to achieve its desired goals (Chang, 2017).

The earliest welfare game originated from the relationship between the government and parolees: the government is willing to grant parole, hoping that the parolee's behavior after release will meet its requirements. In turn, the inmate must demonstrate behavior that meets the government's expectations during incarceration to

obtain the chance of parole. The government is willing to release a particular prisoner if it believes the parolee has a high chance of successful rehabilitation according to its standards. However, once granted parole, the parolee might become complacent and revert to previous behavior.

The welfare game may appear to lack a stable Nash equilibrium. However, in reality, the formation of Nash equilibrium depends on whether the conditions tied to both parties' strategies meet the expectations of the other party. Therefore, each side of the game attempts to predict the other's behavior to determine their own best course of action.

### 3. Game Theory Derivation

#### 3.1 The background of game

Russia's invasion of Ukraine has not slowed down, and NATO, led by the United States, along with EU member states, have been assisting Ukraine to the best of their ability. Economically, as of April 2023, the U.S. has provided financial aid totaling 1.08 trillion TWD (Sanli iNEWS, 2023), while the UK has pledged to offer \$50 billion in loans to help Ukraine mitigate the economic impact of the war.(GOV.UK, 2022) France has provided €30 billion in aid(Xinhua News, 2022), and Canada, through the issuance of a \$50 billion five-year bond, has transferred the proceeds via the International Monetary Fund to Ukraine to support essential services and energy infrastructure.(Canada.ca, 2022) Germany has contributed €2 billion for security capacity building, which is used to reimburse EU member states for their military support to Ukraine.(Kyiv post, 2022 C) Various countries have provided financial assistance to Ukraine through different means to help its people maintain a semblance of normal life during the war.

To acquire the necessary equipment to counter the Russian invasion, Ukrainian President Zelensky has made international appeals through the media, calling for more weapons and ammunition to defend against Russia's aggression. His plea has been met with positive responses from many countries worldwide, which have provided military aid. For example, the United States, through the Department of Defense, has transferred equipment to Ukraine, including Stinger air defense systems, Javelins, light anti-armor weapons, AT-4 anti-armor systems, tactical unmanned aerial systems, grenade launchers, mortar shells, bulletproof vests, anti-drone tracking radars, and High Mobility Multipurpose Wheeled Vehicles, all aimed at helping Ukraine defend its territory.(The White House, 2022) The UK has trained Ukrainians to use underwater drones to clear coastal mines and has provided logistical support such as electronic warfare devices, night vision devices, GPS jamming systems, counter-artillery radar systems, and heavy unmanned aerial systems (Kyiv post, 2022 A, D).

While EU member states were still discussing the specifics of their aid, Poland had already supplied T-72 tanks, multiple rocket launchers, self-propelled howitzers, surveillance drones, and infantry fighting vehicles, along with transporting oil to ensure Ukraine's vehicles had sufficient fuel. Poland has become the largest donor among EU member states due to its significant contributions (World Journal, 2022).

Amid ongoing aid from various countries, on May 26, a Ukrainian military commander expressed on Telegram: "We urgently need weapons capable of hitting the enemy from a distance. Delays should not occur, as the cost of waiting is the lives of our people, who are defending the world from Russia's attacks." As the Russia-Ukraine war continues, the relationship between Ukraine and the Western allies has evolved into a dynamic where the aid-seeking Ukraine relies on the timely assistance of the donors. If the providers delay their aid, Ukraine will struggle to continue resisting Russia's invasion (Kyiv post, 2022 B).

### 3.2 Game information explanation

- **Participants:** One side consists of Western countries (the U.S. and Europe), and the other side is Ukraine.
- **Information Status**

**Complete Information:** Both Western countries and Ukraine are fully aware that resisting Russia's invasion requires significant expenditure of economic resources and military weapons, thus constituting a complete information scenario.

**Information Symmetry:** Both sides share the same understanding of the threat posed by Russia's invasion and potential annexation of Ukraine, which could destabilize Europe and affect geopolitical power. This represents symmetric information, as both parties possess essential information regarding their mutual needs.

### 3.3 Explanation of the payoff functions

#### 1. Western countries do not provide aid, and Ukraine does not resist Russia's invasion:

Let  $\Pi$  represent the baseline payoff for Western countries before providing aid to Ukraine, which is positive.  $\Omega$  represents Ukraine's negative payoff for resisting Russia's invasion without any economic or military aid from the West. In this case, both parties' baseline payoffs are  $(0, 0)$ .

#### 2. Western countries do not provide aid, but Ukraine resists Russia's invasion:

Russia's invasion continues unabated, and Ukraine's people, in defense of their territory, ethnic survival, and the values of freedom and democracy (denoted by " $c$ "), vow to resist the Russian invasion to the death. Ukraine's payoff is  $\Omega+c$ , where  $\Omega$  represents Ukraine's losses in resisting Russia (a negative value). Meanwhile, Western countries, unaware of the full value of Ukraine's defense of its land and democratic values, act only in their own interest and provide no aid. If Ukraine fails, it would threaten the

security of other European countries and disturb the geopolitical balance of power. Thus, the Western countries' payoff is  $\pi-c$ .

**3. Western countries provide aid, and Ukraine resists Russia's invasion:**

Ukraine, under heavy attack from Russia and facing indiscriminate missile strikes on cities, continues to resist, despite a lack of sufficient resources and weapons. The payoff for Ukraine is  $\Omega+k$ . At the same time, seeing Ukraine's strong will to fight and recognizing the need to protect other European countries from similar threats, the Western countries offer economic and military aid to Ukraine. This helps maintain the geopolitical balance and defend the values of freedom and democracy. The payoff for the Western countries rises to  $\pi+c-k$ .

**4. Western countries provide aid, but Ukraine does not resist Russia's invasion:**

Suppose Ukraine receives significant economic and military aid from the Western countries. However, due to corruption and misuse of funds by oligarchs, the aid fails to reach the front lines, weakening the fighting effectiveness of the Ukrainian army and threatening the security of other European nations and the geopolitical balance. In this case, the payoff for the Western countries drops to  $\pi-k$ . Meanwhile, despite receiving substantial aid, corruption among Ukrainian officials may divert resources for personal gain, forcing the army to rely solely on a limited amount of high-quality Western weaponry to resist the Russian invasion. Thus, Ukraine's payoff increases to  $\Omega+c+k$ .

Table1  
The Welfare game matrix between US-NATO and Ukraine

Payoff ( US-NATO, Ukraine)		Ukraine	
		Anti-invasion ( $q$ )	Non anti-invasion ( $1-q$ )
US-NATO	Support ( $p$ )	$(\pi+c-k, \Omega+k)$	$(\pi-k, \Omega+c+k)$
	Non-support ( $1-p$ )	$(\pi-c, \Omega+c)$	$(0, 0)$

**3.4 Game Analysis**

From the perspective of Western countries' decision-making, whether Ukraine resists the Russian invasion or not will influence their strategy. In other words, if aiding Ukraine yields a better payoff than not aiding, Western countries will provide economic, military, and humanitarian assistance to support Ukraine in resisting Russia, thereby preventing Russia from altering the European power landscape. Conversely, if the payoff is lower, they

will refrain from assisting Ukraine. From Ukraine's perspective, its strategic decision depends on whether it can successfully secure recognition and aid from Western countries. Otherwise, it will face isolation. Below is an analysis of the payoffs under different strategy combinations:

### 1. Initial Dynamic Behavior and Conditions in This Welfare Game from the Perspectives of Ukraine and Western Countries:

- (1) When Western allies prefer not to aid Ukraine, and Ukraine prefers to fully resist Russia to garner international sympathy:

$$\Omega + c > 0, \quad c > -\Omega \quad (1)$$

- (2) When Ukraine resists Russia's invasion, and Western allies prefer to aid Ukraine to gain international political advantage and weaken Russia:

$$\because \pi + c - k > \pi - c \quad \therefore c > \frac{1}{2}k, \text{ or } k < 2c \quad (2)$$

This means that Western allies prefer to aid Ukraine when the value of military and economic support ( $k$ ) they provide is not less than twice the value of Ukraine's efforts to defend its territory, ethnic survival, and democratic values ( $c$ ).

- (3) When Western allies prefer to provide unlimited aid to Ukraine, and Ukraine prefers not to fully resist Russia, aiming to gain economic benefits resulted from  $\Omega + c + k > \Omega + k$ .

- (4) When Ukraine prefers not to fully resist Russia, and Western allies prefer not to aid Ukraine:

$$\because \pi + c - k > \pi - c \quad \therefore c > \frac{1}{2}k, \text{ or } k < 2c \quad (3)$$

This implies that when the value of the aid Western allies provide exceeds their original benefit, they will abandon their support for Ukraine.

### 2. From the Perspective of Western Countries

Analyzing the situation from the standpoint of Western countries, if they provide economic and military aid, their decision criterion will be based on maximizing total payoffs:

$$\begin{aligned} \pi_E &= p[q(\pi + c - k) + (1 - q)(\pi - k)] + (1 - p)[q(\pi - c)] \\ &= pq(2c - \pi) + p(\pi - k) + q(\pi - c) \end{aligned} \quad (4)$$

To solve for the maximization of total payoffs, we differentiate equation (4) and set it to zero:

$$\frac{d\pi_E}{dp} = q(2c - \pi) + (\pi - k) = 0 \quad (5)$$

$$\Rightarrow q = \frac{k - \pi}{2c - \pi}$$

From equation (5), it is evident that the larger the value of  $c$  (the Ukrainian people's resolve to defend their territory, ethnic survival, and democratic values), the smaller the incentive for Ukraine to resist the Russian invasion ( $q$ ). Conversely, the greater the value of Western aid ( $k$ ) or the original interests of the Western alliance ( $\pi$ ), the greater the incentive for Ukraine to resist the Russian invasion ( $q$ ).

Additionally, equation (4) must satisfy the condition that the second derivative is less than zero:

$$\frac{d\pi_E^2}{dp \cdot dq} = 2c - \pi > 0 \quad (6)$$

$$\Rightarrow c > \frac{\pi}{2}$$

This is the necessary condition, meaning that the Ukrainian people's resolve to defend their territory, ethnic survival, and democratic values ( $c$ ) must exceed half of the original interests of the Western alliance ( $\pi$ ). Only then will the Western alliance have the resolve to assist Ukraine in resisting the Russian invasion.

### 3. From the Perspective of Ukraine

Analyzing from Ukraine's perspective, the decision criterion for determining whether to resist the Russian invasion is based on maximizing total payoffs:

$$\begin{aligned} \pi_U &= q[p(\Omega + k) + (1 - p)(\Omega + c)] + (1 - q)[p(\Omega + c + k)] \\ &= q[\Omega + c - p(2c + \Omega)] + p(\Omega + c + k) \end{aligned} \quad (7)$$

To solve for the maximization of total payoffs, we differentiate equation (7) and set it to zero:

$$\frac{\pi_U}{dq} = \Omega + c - p(2c + \Omega) \quad (8)$$

$$p = \frac{c + \Omega}{2c + \Omega}$$

From equation (7), we can see that the greater the losses Ukraine suffers from facing the war against Russia alone ( $\Omega$ ), the smaller the likelihood of receiving aid from Western allies ( $p$ ).

Additionally, equation (7) must satisfy the condition that the second derivative is greater than zero:

$$\frac{d\pi_U^2}{dq \cdot dp} = -(2c + \Omega) < 0, \text{ or } 2c + \Omega > 0 \quad (9)$$

Thus, equation (10) follows:

$$c > \frac{-\Omega}{2} \quad (10)$$

This is the necessary condition, meaning that the Ukrainian people's resolve to defend their territory, ethnic survival, and democratic values ( $c$ ) must exceed half of the losses Ukraine would suffer from facing the war against Russia alone ( $\Omega$ ). Only then will Ukraine have the determination to resist the Russian invasion.

#### 4. Conclusion

Russia's invasion of Ukraine has shown no signs of slowing down. While NATO, led by the United States, and the EU member states seem to be doing everything possible to aid Ukraine, international political and military actions are ultimately driven by national interests. The assistance provided by the Western alliance to Ukraine in resisting Russia's invasion is largely motivated by the goal of weakening Russia's economic and military power.

According to the game theory analysis, the determination of the Western alliance to support Ukraine in resisting Russia's invasion only materializes when the Ukrainian people's resolve to defend their territory, ethnic survival, and democratic values exceeds half of the original interests of the Western alliance. Furthermore, as the Ukrainian people's determination to defend these values increases, the incentive for Ukraine to resist the Russian invasion paradoxically decreases. On the other hand, the greater the value of Western aid ( $k$ ) or the Western alliance's original interests, the stronger Ukraine's motivation to resist the Russian invasion.

From Ukraine's perspective, the determination to resist Russia only arises when the Ukrainian people's resolve to defend their territory, ethnic survival, and democratic values exceeds half of the losses they would face by confronting Russia alone. At the same time, as the losses Ukraine incurs from fighting Russia alone increase, the likelihood of receiving aid from Western allies decreases.

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